International Journal of Engineering Sciences & Research Technology

(A Peer Reviewed Online Journal)
Impact Factor: 5.164

IJESRT

ISSN: 2277-9655
Impact Factor: 5.164
CODEN: IJESS7

IJESRT

INTERNATIONAL JOURNAL OF ENGINEERING SCIENCES & RESEARCH TECHNOLOGY

F_1 -NEIGHBORHOOD AND SQUARE NEIGHBORHOOD DAKSHAYANI INDICES OF SOME NANOSTRUCTURES

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DOI: 10.5281/zenodo.3377432

ABSTRACT

We propose the modified first and second neighborhood Dakshayani indices, F_1-neighborhood Daksiyani index, minus neighborhood Dakshayani index and square neighborhood Dakshayani index of a graph. In this study, we compute the F_1 neighborhood Dakshayani index, minus neighborhood Dakshayani index, square neighborhood Dakshayani index and their polynomials of line graphs of subdivision graphs of 2-D lattice, nanotube, nanotorus of TUC_4C_8 [p, q]. Furthermore we determine the modified first and second neighborhood Dakshayani indices of 2-D lattice, nanotube, nanotorus of TUC_4C_8 [p, q].

Mathematics Subject Classification : 05*C*07, 05*C*12, 05*C*76

KEYWORDS: *modified neighborhood Dakshayani indices, F1-neighborhood Dakshayani index, minus and square neighborhood Dakshayani indices, nanostructure*.

1. INTRODUCTION

A molecular graph is a simple graph such that its vertices correspond to the atoms and the edges to the bonds. Chemical Graph Theory is a branch of Mathematical Chemistry, which has an important effect on the development of Chemical Sciences. In Mathematical Chemistry, topological indices have been found to be useful in discrimination, chemical documentation, QSPR, QSAR and pharmaceutical drug design. There has been considerable interest in the general problem of determining topological indices, see [1, 2].

Let G be a finite, simple, connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(u)$ of a vertex u is the number of vertices adjacent to u. The edge connecting the vertices u and v will be denoted by uv. The subdivision graph $S(G)$ is the graph obtained from G by replacing each of its edges by a path of length two. The line graph $L(G)$ of G is the graph whose vertex set corresponds to the edges of G such that two vertices of $L(G)$ are adjacent if the corresponding edges of G are adjacent.

Let $N_G(v) = \{ u : uv \in E(G)\}$. Let $D_G(v) = d_G(v) + \sum_{u \in N_G(v)} d_G(u)$ is the degree sum of closed neighborhood vertices of v. For other graph terminology and notation, refer [3].

We need the following results.

Lemma 1. Let G be a (p, q) graph. Then $S(G)$ has $p+q$ vertices and $2q$ edges.

Lemma 2. Let G be a (p, q) graph. Then $L(G)$ has q vertices and $\frac{1}{2}\sum_{i=1}^{p} d_G(u_i)^2 - q$ edges.

Recently the modified vertex neighborhood Dakshayani index of a graph is defined as [4]

$$ {}^{m}ND_v(G) = \sum_{u \in V(G)} \frac{1}{D_G(u)^2}. $$

We now introduce the modified first and second neighborhood Dakshayani indices of a graph, defined as

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$$ {}^{m}ND_1(G)=\sum_{uv\in E(G)}\frac{1}{D_G(u)+D_G(v)} $$

(1)

$$ {}^{m}ND_2(G)=\sum_{uv\in E(G)}\frac{1}{D_G(u)D_G(v)}. $$

(2)

In [4], the *F*-neighborhood Dakshayani index of *G* was introduced by Kulli, defined as

$$ FND(G)=\sum_{u\in V(G)}D_G(u)^3. $$

Now we propose the F_1-neighborhood Dakshayani index of a graph *G* and it is defined as

$$ F_1ND(G)=\sum_{uv\in E(G)}\left[D_G(u)^2+D_G(v)^2\right]. $$

(3)

In [5], Albertson introduced the irregularity index (called as minus index [6]) and defined it as

$$ M_i(G)=\sum_{uv\in E(G)}\left|d_G(u)-d_G(v)\right|. $$

Recently, the square ve-*d*egree index was proposed by Kulli [7] and defined it as

$$ Q_{ve}(G)=\sum_{uv\in E(G)}\left[d_{ve}(u)-d_{ve}(v)\right]^2. $$

We introduce the minus neighborhood Dakshayani index and square neighborhood Dakshayani index of *G*, defined as

$$ MND(G)=\sum_{uv\in E(G)}\left|D_G(u)-D_G(v)\right|. $$

(4)

$$ QND(G)=\sum_{uv\in E(G)}\left[D_G(u)-D_G(v)\right]^2. $$

(5)

Recently, some square indices were proposed and studied such as square Revan index [8], square *KV* index [9], square reverse index [10], square leap index [11], square *F*-index [12].

Considering the F_1 neighborhood Dakshayani index, minus neighborhood Dakshayani index and square neighborhood Dakshayani index, we introduce the F_1 neighborhood Dakshayani polynomial, minus neighborhood Dakshayani polynomial and square neighborhood Dakshayani polynomial of a graph as

$$ F_1ND(G,x)=\sum_{uv\in E(G)}x^{\left[D_G(u)^2+D_G(v)^2\right]}. $$

(6)

$$ MND(G,x)=\sum_{uv\in E(G)}x^{\left|D_G(u)-D_G(v)\right|}. $$

(7)

$$ QND(G,x)=\sum_{uv\in E(G)}x^{\left[D_G(u)-D_G(v)\right]^2}. $$

(8)

Let *p* and *q* denote the number of squares in a row and the number of rows of squares respectively in 2-*D* lattice, nanotube and nanotorus of TUC_4C_8 [*p*, *q*]. The 2-*D* lattice, nanotube and nanotorus of TUC_4C_8[4, 2] are

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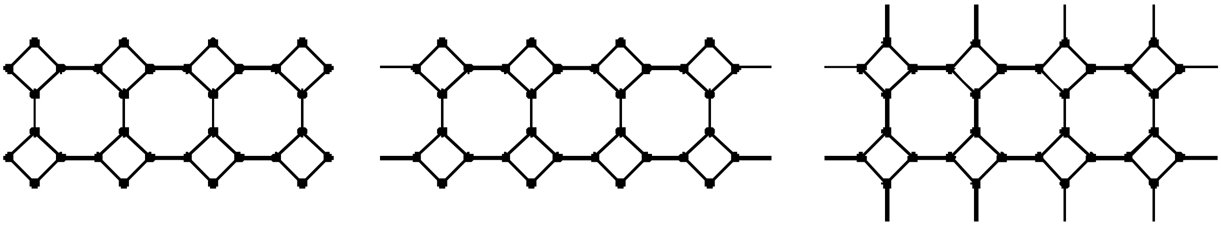
presented in Figure 1 (a), (b), (c) respectively, Some study on these nanostructures can be found in [13, 14, 15, 16, 17, 18].

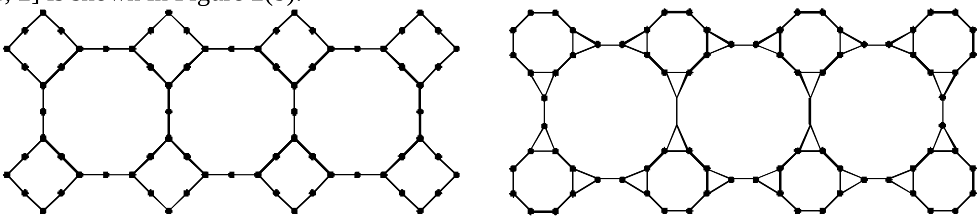


(a)2D-lattice of $TUC_4C_8[4, 2]$ (b) $TUC_4C_8[4,2]$ nanotube (c) TUC_4C_8 [4, 2] nanotorus

Figure 1

2. 2-*D* lattice of TUC_4C_8 [*p, q*]

We consider 2-*D* lattice of TUC_4C_8 [*p*, *q*] nanostructures. The line graph of subdivision graph of 2-D lattice of TUC_4C_8 [4, 2] is shown in Figure 2(b).



(a) Subdivision graph of 2-*D* lattice of $TUC_4C_8[4, 2]$

(b) Line graph of subdivision graph of 2-*D* lattice of $TUC_4C_8[4, 2]$

Figure 2

Let *G* be the line graph of subdivision graph of 2-*D* lattice of $TUC_4C_8[p, q]$. The 2-*D* lattice of TUC_4C_8 [*p*, *q*] is a graph with $4pq$ vertices and $6pq – p – q$ edges. By Lemma 1, the subdivision graph of 2-*D* lattice of TUC_4C_8 [*p*, *q*] is a graph with $10 pq – p – q$ vertices and $2 (6pq – p – q)$ edges. Thus by Lemma 2, *G* has $2 (6pq – p – q)$ vertices and $18 pq – 5p – 5q$ edges. Thus the edge partition of *G* based on the degree sum of closed neighborhood vertices of each vertex is obtained as given in Table 1 and Table 2.

Table 1. Edge partition of G when p > 1, q > 1

$D_G(u), D_G(v)\backslash uv \in E(G)$	(6,6)	(6,7)	(7, 7)	(7,11)	(11, 12)	(12, 12)
Number of edges	4	8	$2(p+q–4)$	$4(p+q–2)$	$8 (p+q–2)$	$2 (9pq+10) –19 (p+q)$

Table 2. Edge partition of G when p > 1, q = 1

$D_G(u), D_G(v)\backslash uv \in E(G)$	(6,6)	(6,7)	(7, 7)	(7,11)	(11, 11)	(11, 12)	(12, 12)
Number of edges	6	4	$2(p–2)$	$4(p–1)$	$2 (p–1)$	$4(p–1)$	$(p–1)$

Theorem 1. Let *G* be the line graph of subdivision graph of 2-*D* lattice of TUC_4C_8 [*p*, *q*]. Then

$$^{m}ND_1(G) = \frac{3}{4}pq + \left(\frac{1}{7}+\frac{2}{9}+\frac{8}{13}-\frac{19}{24}\right)(p+q) + \left(\frac{1}{3}+\frac{8}{13}-\frac{4}{7}-\frac{4}{9}-\frac{16}{23}+\frac{5}{6}\right), \text{ if } p > 1, q > 1,$$

$$= \left(\frac{1}{7}+\frac{2}{9}+\frac{1}{11}+\frac{4}{23}+\frac{1}{24}\right)p + \left(\frac{1}{2}+\frac{4}{13}-\frac{2}{7}-\frac{2}{9}-\frac{1}{11}-\frac{4}{23}-\frac{1}{24}\right), \text{ if } p > 1, q = 1.$$

Proof: Case 1. Suppose $p > 1, q > 1$.

From the definition of the modified first neighborhood Dakshayani index and by using Table 1, we obtain

$$^{m}ND_1(G) = \sum_{uv\in E(G)} \frac{1}{D_G(u)+D_G(v)}$$

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Impact Factor: 5.164
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$$=\left(\frac{1}{6+6}\right)4+\left(\frac{1}{6+7}\right)8+\left(\frac{1}{7+7}\right)2(p+q-4)+\left(\frac{1}{7+11}\right)4(p+q-2)$$
$$+\left(\frac{1}{11+12}\right)8(p+q-2)+\left(\frac{1}{12+12}\right)\left[2(9pq+10)-19(p+q)\right]$$
$$=\frac{3}{4}pq+\left(\frac{1}{7}+\frac{2}{9}+\frac{8}{13}-\frac{19}{24}\right)(p+q)+\left(\frac{1}{3}+\frac{8}{13}-\frac{4}{7}-\frac{4}{9}-\frac{16}{23}+\frac{5}{6}\right)$$

Case 2. Suppose $p > 1$, $q = 1$.
From the definition of the modified first neighborhood Dakshayani index and by using Table 2, we deduce

$$^{m}ND_2(G)=\sum_{uv\in E(G)}\frac{1}{D_G(u)+D_G(v)}$$
$$=\left(\frac{1}{6+6}\right)6+\left(\frac{1}{6+7}\right)4+\left(\frac{1}{7+7}\right)2(p-2)+\left(\frac{1}{7+11}\right)4(p-1)+\left(\frac{1}{11+11}\right)2(p-1)$$
$$+\left(\frac{1}{11+12}\right)4(p-1)+\left(\frac{1}{12+12}\right)(p-1)$$
$$=\left(\frac{1}{7}+\frac{2}{9}+\frac{1}{11}+\frac{4}{23}+\frac{1}{24}\right)p+\left(\frac{1}{2}+\frac{4}{13}-\frac{2}{7}-\frac{2}{9}-\frac{1}{11}-\frac{4}{23}-\frac{1}{24}\right).$$

Theorem 2. Let G be the line graph of subdivision graph of 2-D lattice of $TUC_4C_8\,[p, q]$. Then

$$^{m}ND_2(G)=\frac{1}{8}pq+\left(\frac{2}{49}+\frac{4}{77}+\frac{2}{33}-\frac{19}{144}\right)(p+q)+\left(\frac{1}{9}+\frac{4}{21}-\frac{8}{49}-\frac{8}{77}-\frac{4}{33}+\frac{5}{36}\right),\text{ if } p>1,\ q>1,$$

$$=\left(\frac{2}{49}+\frac{4}{77}+\frac{2}{121}+\frac{1}{33}+\frac{1}{144}\right)p+\left(\frac{1}{6}+\frac{2}{21}-\frac{4}{49}-\frac{4}{77}-\frac{2}{121}-\frac{1}{33}-\frac{1}{144}\right),\text{ if } p>1,\ q=1.$$

Proof: Case 1. Suppose $p > 1$, $q > 1$.
From the definition of the modified second neighborhood Dakshayani index and by using Table 1, we derive

$$^{m}ND_2(G)=\sum_{uv\in E(G)}\frac{1}{D_G(u)D_G(v)}$$
$$=\left(\frac{1}{6\times6}\right)4+\left(\frac{1}{6\times7}\right)8+\left(\frac{1}{7\times7}\right)2(p+q-4)+\left(\frac{1}{7\times11}\right)4(p+q-2)$$
$$+\left(\frac{1}{11\times12}\right)8(p+q-2)+\left(\frac{1}{12\times12}\right)\left[2(9pq+10)-19(p+q)\right]$$
$$=\frac{1}{8}pq+\left(\frac{2}{49}+\frac{4}{77}+\frac{2}{33}-\frac{19}{144}\right)(p+q)+\left(\frac{1}{9}+\frac{4}{21}-\frac{8}{49}-\frac{8}{77}-\frac{4}{33}+\frac{5}{36}\right)$$

Case 2. Suppose $p > 1$, $q = 1$.
By using the definition of the modified second neighborhood Dakshayani index and by using Table 2, we deduce

$$^{m}ND_2(G)=\sum_{uv\in E(G)}\frac{1}{D_G(u)D_G(v)}$$

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$$=\left(\frac{1}{6\times 6}\right)6+\left(\frac{1}{6\times 7}\right)4+\left(\frac{1}{7\times 7}\right)2(p-2)+\left(\frac{1}{7\times 11}\right)4(p-1)$$

$$+\left(\frac{1}{11\times 11}\right)2(p-1)+\left(\frac{1}{11\times 12}\right)4(p-1)+\left(\frac{1}{12\times 12}\right)(p-1)$$

$$=\left(\frac{2}{49}+\frac{4}{77}+\frac{2}{121}+\frac{1}{33}+\frac{1}{144}\right)p+\left(\frac{1}{6}+\frac{2}{21}-\frac{4}{49}-\frac{4}{77}-\frac{2}{121}-\frac{1}{33}-\frac{1}{144}\right)$$

Theorem 3. Let G be the line graph of subdivision graph of 2-D lattice of $TUC_4C_8[p, q]$. Then

$F_1ND(G)$ = 5184 pq – 2476 (p+q) + 344, if $p > 1$, $q > 1$,
= 2708 p – 2132, if $p > 1$, $q = 1$.

Proof: Case 1. Suppose $p > 1$, $q>1$.
From the definition of the F_1 neighborhood Dakshayani index and by using Table 1, we have

$$F_1ND(G)=\sum_{uv\in E(G)}\left[D_G(u)^2+D_G(v)^2\right]$$

$$=(6^2+6^2)4+(6^2+7^2)8+(7^2+7^2)2(p+q-4)+(7^2+11^2)4(p+q-2)$$

$$+(11^2+12^2)8(p+q-2)+(12^2+12^2)\left[2(9pq+10)-10(p+q)\right]$$

$$=5184pq-2476(p+q)+344$$

Case 2. Suppose $p > 1$, $q = 1$.
From the definition of the F_1-neighborhood Dakshayani index and by using Table 2, we obtain

$$F_1ND(G)=\sum_{uv\in E(G)}\left[D_G(u)^2+D_G(v)^2\right]$$

$$=(6^2+6^2)6+(6^2+7^2)4+(7^2+7^2)2(p-2)+(7^2+11^2)4(p-1)$$

$$+(11^2+11^2)2(p-1)+(11^2+12^2)4(p-1)+(12^2+12^2)(p-1)$$

$$= 2708p - 2132.$$

Theorem 4. The minus neighborhood Dakshayani index of the line graph of subdivision graph of 2-D lattice of $TUC_4C_8[p, q]$ is

$MND(G)$ = 24 (p + q) – 40, if $p > 1$ $q > 1$
= 20 p – 16, if $p>1$, $q = 1$.

Proof: Case 1. Suppose p>1, $q > 1$.
From the definition of the minus neighborhood Dakshayani index and by using Table 1, we deduce

$$MND(G)=\sum_{uv\in E(G)}\left|D_G(u)-D_G(v)\right|$$

$$= |6-6|\,4 + |6-7|\,8 + |7-7|\,2(p+q-4) + |7-11|\,4(p+q-2)$$
$$+|11-12|8\,(p+q-2)+ |12-12|\,[2(9pq + 10) - 19\,(p+q)]$$
$$= 24\,(p+q) - 40.$$

Case 2. Suppose p>1, $q = 1$.
From the definition of the minus neighborhood Dakshayani index and by using Table 2, we derive

$$MND(G)=\sum_{uv\in E(G)}\left|D_G(u)-D_G(v)\right|$$

$$= |6-6|\,6 + |6-7|\,4 + |7-7|\,2(p-2) + |7-11|\,4(p-1)$$
$$+|11-11|2(p-1)+|11-12|\,4(p-1) + |12-12|\,(p-1)$$
$$= 20p - 16.$$

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Theorem 5. Let G be the line graph of subdivision graph of 2-D lattice of TUC_4C_8 [p, q]. Then

$$QND(G) = \begin{cases} 72(p+q) - 144, & \text{if } p > 1, q > 1. \\ 68p - 64, & \text{if } p > 1, q = 1. \end{cases}$$

Proof: Case 1. Suppose $p > 1$, $q > 1$.
By using the definition of the square neighborhood Dakshayami index and Table 1, we deduce

$$QND(G) = \sum_{uv\in E(G)} \left[D_G(u) - D_G(v)\right]^2$$
$$= (6-6)^2 4 + (6-7)^2 8 + (7-7)^2 2(p+q-4) + (7-11)^2 4(p+q-2)$$
$$+ (11-12)^2 8(p+q-2) + (12-12)^2 \left[2(9pq+10) - 196(p+q)\right]$$
$$= 72(p+q) - 144.$$

Case 2. Suppose $p>1$, $q = 1$.
From the definition of the square neighborhood Dakshayani index and by using Table 2, we derive

$$QND(G) = \sum_{uv\in E(G)} \left[D_G(u) - D_G(v)\right]^2$$
$$= (6-6)^2 6 + (6-7)^2 4 + (7-7)^2 2(p-2) + (7-11)^2 4(p-1)$$
$$+ (11-11)^2 2(p-1) + (11-12)^2 4(p-1) + (12-12)^2 (p-1)$$
$$= 68p - 64.$$

Theorem 6. Let G be the line graph subdivision graph of 2-D lattice of TUC_4C_8 [p, q]. Then

$$F_1ND(G,x) = 4x^{72} + 8x^{85} + 2(p+q-4)x^{98} + 4(p+q-2)x^{178} + 8(p+q-2)x^{265}$$
$$+ \left[2(9pq+10) - 19(p+q)\right]x^{288}, \quad \text{if } p>1, q>1,$$
$$= 6x^{72} + 4x^{85} + 2(p-2)x^{98} + 4(p-1)x^{178} + 2(p-1)x^{242}$$
$$+ 4(p-1)x^{265} + (p-1)x^{288}, \quad \text{if } p>1, q=1.$$

Proof: Case 1. Suppose $p>1$, $q > 1$.
From equation (6) and by using Table 1, we deduce

$$F_1ND(G,x) = \sum_{uv\in E(G)} x^{D_G(u)^2 + D_G(v)^2}$$
$$= 4x^{6^2+6^2} + 8x^{6^2+7^2} + 2(p+q-4)x^{7^2+7^2} + 4(p+q-2)x^{7^2+11^2}$$
$$+ 8(p+q-2)x^{11^2+12^2} + \left[2(9pq+10) - 19(p+q)\right]x^{12^2+12^2}$$
$$= 4x^{72} + 8x^{85} + 2(p+q-4)x^{98} + 4(p+q-2)x^{178}$$
$$+ 8(p+q-2)x^{265} + \left[2(9pq+10) - 19(p+q)\right]x^{288}.$$

Case 2. Suppose p>1, $q = 1$.
From equation (6) and by using Table 2, we derive

$$F_1ND(G,x) = \sum_{uv\in E(G)} x^{D_G(u)^2 + D_G(v)^2}$$
$$= 6x^{6^2+6^2} + 4x^{6^2+7^2} + 2(p-2)x^{7^2+7^2} + 4(p-1)x^{7^2+11^2}$$

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Impact Factor: 5.164
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$$+2(p-1)x^{11^2+11^2}+4(p-1)x^{11^2+12^2}+(p-1)x^{12^2+12^2}$$
$$=6x^{72}+4x^{85}+2(p-2)x^{98}+4(p-1)x^{178}$$
$$+2(p-1)x^{242}+4(p-1)x^{265}+(p-1)x^{288}.$$

Theorem 7. Let G be the line graph of subdivision graph of 2-D lattice of TUC_4C_8 [p, q]. Then

$$MND(G,x)=\left[18pq-17(p+q)+16\right]x^0+8(p+q-1)x^1+4(p+q-2)x^4, \quad \text{if } p>1,\ q>1,$$
$$=(5p-1)x^0+4px^1+4(p-1)x^4, \quad \text{if } p>1,\ q=1.$$

Proof: Case 1. Suppose $p>1$, $q>1$.
From equation (7) and by using Table 1, we obtain

$$MND(G,x)=\sum_{uv\in E(G)} x^{|D_G(u)-D_G(v)|}$$
$$=4x^{|6-6|}+8x^{|6-7|}+2(p+q-4)x^{|7-7|}+4(p+q-2)x^{|7-11|}$$
$$+8(p+q-2)x^{|11-12|}+\left[2(9pq+10)-19(p+q)\right]x^{|12-12|}$$
$$=\left[18pq-17(p+q)+16\right]x^0+8(p+q-1)x^1+4(p+q-2)x^4.$$

Case 2. Suppose $p>1$, $q=1$.
From equation (7) and by using Table 2, we have

$$MND(G,x)=\sum_{uv\in E(G)} x^{|D_G(u)-D_G(v)|}$$
$$=6x^{|6-6|}+4x^{|6-7|}+2(p-2)x^{|7-7|}+4(p-1)x^{|7-11|}$$
$$+2(p-1)x^{|11-11|}+4(p-1)x^{|11-12|}+(p-1)x^{|12-12|}$$
$$=(5p-1)x^0+4px^1+4(p-1)x^4.$$

Theorem 8. Let G be the line graph of subdivision graph of 2-D lattice of TUC_4C_8 [p, q]. Then

$$QND(G,x)=\left[18pq-17(p+q)+16\right]x^0+8(p+q-1)x^1+4(p+q-2)x^{16}, \quad \text{if } p>1,\ q>1,$$
$$=(5p-1)x^0+4px^1+4(p-1)x^{16}, \quad \text{if } p>1,\ q=1.$$

Proof: Case 1. Suppose $p>1$, $q>1$.

By using equation (8) and Table 1, we deduce

$$QND(G,x)=\sum_{uv\in E(G)} x^{\left[D_G(u)-D_G(v)\right]^2}$$
$$=4x^{(6-6)^2}+8x^{(6-7)^2}+2(p+q-4)4x^{(7-7)^2}+4(p+q-2)4x^{(7-11)^2}$$
$$+8(p+q-2)x^{(11-12)^2}+\left[2(9pq+10)-19(p+q)\right]x^{(12-12)^2}$$
$$=\left[18pq-17(p+q)+16\right]x^0+8(p+q-1)x^1+4(p+q-2)x^{16}.$$

Case 2. Suppose $p>1$, $q=1$.

By using equation (9) and Table 2, we obtain

ISSN: 2277-9655
Impact Factor: 5.164
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$$QND(G,x) = \sum_{uv \in E(G)} x^{\left[D_G(u)-D_G(v)\right]^2}$$

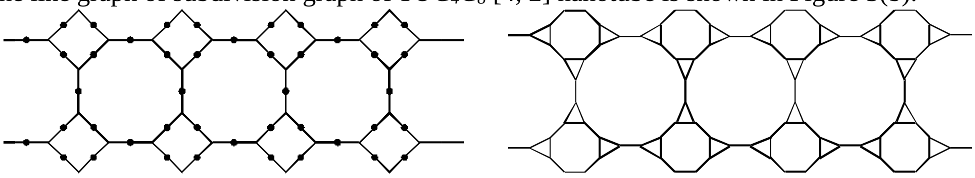
$$= 6x^{(6-6)^2} + 4x^{(6-7)^2} + 2(p-2)4x^{(7-7)^2} + 4(p-1)x^{(7-11)^2}$$

$$+2(p-1)x^{(11-11)^2} + 4(p-1)x^{(11-12)^2} + (p-1)x^{(12-12)^2}.$$

$$= (5p-1)x^0 + 4px^1 + 4(p-1)x^{16}.$$

3. *TUC_4C_8* [*p*, *q*] Nanotube

The line graph of subdivision graph of TUC_4C_8 [4, 2] nanotube is shown in Figure 3(b).



(a)
Subdivision graph
of TUC_4C_8[4, 2] nanotube

(b)
Line graph of subdivision
graph of TUC_4C_8[4, 2] nanotube

Figure 3

Let *H* be the line graph of subdivision graph of TUC_4C_8 [*p*, *q*] nanotube. The TUC_4C_8 [*p*, *q*] nanotube is a graph with 4 *pq* vertices and 6 *pq* – *p* edges. By Lemma 1, the subdivision graph of TUC_4C_8 [*p*, *q*] nanotube is a graph with 10*pq* – *p* vertices and 12*pq* – 2*p* edges. Thus by Lemma 2, *H* has 12*pq* – 2*p* vertices and 18*pq* – 5*p* edges. The vertices of *H* are either of degree 2 or 3, see Figure 3(b). Therefore the partition of the edge set of *H* is as given in Table 3 and Table 4.

Table 3. Edge partition of H when p >1, q >1

$D_G(u), D_G(v)\backslash uv \in E(H)$	(7, 7)	(7,11)	(11,12)	(12, 12)
Number of edges	2*p*	4*p*	8*p*	18*pq* – 19*p*

Table 4. Edge partition of H when p > 1, q = 1

$D_G(u), D_G(v)\backslash uv \in E(H)$	(7, 7)	(7, 11)	(11, 11)	(11, 12)	(12, 12)
Number of edges	2*p*	4*p*	2*p*	4*p*	*p*

Theorem 9. Let *H* be the line graph of subdivision graph of TUC_4C_8[*p*, *q*] nanotube. Then

$$^{m}ND_1(H) = \frac{3}{4}pq + \left(\frac{1}{7}+\frac{2}{9}+\frac{8}{23}-\frac{19}{24}\right)p, \qquad \text{if } p > 1,\ q > 1,$$

$$= \left(\frac{1}{7}+\frac{2}{9}+\frac{1}{11}+\frac{4}{23}+\frac{1}{24}\right)p, \qquad \text{if } p > 1,\ q = 1.$$

Proof: Case 1. Suppose *p* > 1, *q* >1.
By using equation (1) and Table 3, we have

$$^{m}ND_1(H) = \sum_{uv \in E(H)} \frac{1}{D_H(u)+D_H(v)}$$

$$= \left(\frac{1}{7+7}\right)2p + \left(\frac{1}{7+11}\right)4p + \left(\frac{1}{11+12}\right)8p + \left(\frac{1}{12+12}\right)(18pq-19p)$$

$$= \frac{3}{4}pq + \left(\frac{1}{7}+\frac{2}{9}+\frac{8}{23}-\frac{19}{24}\right)p.$$

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Impact Factor: 5.164
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Case 2. Suppose $p > 1$, $q = 1$.
From equation (1) and by using Table 4, we obtain

$$
\begin{aligned}
{}^{m}ND_1(H) &= \sum_{uv \in E(H)} \frac{1}{D_H(u)+D_H(v)} \\
&= \left(\frac{1}{7+7}\right)2p + \left(\frac{1}{7+11}\right)4p + \left(\frac{1}{11+11}\right)2p + \left(\frac{1}{11+12}\right)4p + \left(\frac{1}{12+12}\right)p \\
&= \left(\frac{1}{7} + \frac{2}{9} + \frac{1}{11} + \frac{4}{23} + \frac{1}{24}\right)p.
\end{aligned}
$$

Theorem 10. Let H be the line graph of subdivision graph of $TUC_4C_8\,[p, q]$ nanotube. Then

$$
\begin{aligned}
{}^{m}ND_2(H) &= \frac{1}{8}pq + \left(\frac{2}{49} + \frac{4}{77} + \frac{8}{132} - \frac{19}{144}\right)p, \quad \text{if } p > 1, q > 1, \\
&= \left(\frac{2}{49} + \frac{4}{77} + \frac{2}{121} + \frac{4}{132} + \frac{1}{144}\right)p, \quad \text{if } p > 1, q = 1.
\end{aligned}
$$

Proof: Case 1. Suppose $p > 1$, $q > 1$.
From equation (2) and by using Table 1, we have

$$
\begin{aligned}
{}^{m}ND_2(H) &= \sum_{uv \in E(H)} \frac{1}{D_H(u)D_H(v)} \\
&= \left(\frac{1}{7\times 7}\right)2p + \left(\frac{1}{7\times 11}\right)4p + \left(\frac{1}{11\times 12}\right)8p + \left(\frac{1}{12\times 12}\right)(18pq - 19p) \\
&= \frac{1}{8}pq + \left(\frac{2}{49} + \frac{4}{77} + \frac{8}{132} - \frac{19}{144}\right)p.
\end{aligned}
$$

Case 2. Suppose $p > 1$, $q = 1$.
By using equation (2) and Table 2, we deduce

$$
\begin{aligned}
{}^{m}ND_2(H) &= \sum_{uv \in E(H)} \frac{1}{D_H(u)D_H(v)} \\
&= \left(\frac{1}{7\times 7}\right)2p + \left(\frac{1}{7\times 11}\right)4p + \left(\frac{1}{11\times 11}\right)2p + \left(\frac{1}{11\times 12}\right)4p + \left(\frac{1}{12\times 12}\right)p \\
&= \left(\frac{2}{49} + \frac{4}{77} + \frac{2}{121} + \frac{4}{132} + \frac{1}{144}\right)p.
\end{aligned}
$$

Theorem 11. Let H be the line graph of subdivision graph of $TUC_4C_8\,[p, q]$ nanotube. Then

$$
\begin{aligned}
F_1ND(H) &= 5184pq - 2476p, \quad \text{if } p > 1, q > 1, \\
&= 2708p, \quad \text{if } p > 1, q = 1.
\end{aligned}
$$

Proof: Case 1. Suppose $p > 1$, $q > 1$.
By using equation (3) and Table 3, we deduce

$$
\begin{aligned}
F_1ND(H) &= \sum_{uv \in E(H)} \left[D_H(u)^2 + D_H(v)^2\right] \\
&= (7^2+7^2)2p + (7^2+11^2)4p + (11^2+12^2)8p + (12^2+12^2)(18pq - 19p)
\end{aligned}
$$

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$$= 5184pq - 2476p.$$

Case 2. Suppose $p > 1$, $q = 1$.
From equation (3) and by using Table 4, we derive

$$F_1ND(H) = \sum_{uv \in E(H)} \left[D_H(u)^2 + D_H(v)^2 \right]$$
$$= (7^2+7^2)2p + (7^2+11^2)4p + (11^2+11^2)2p + (11^2+12^2)4p + (12^2+12^2)p$$
$$= 2708p.$$

Theorem 12. Let H be the line graph subdivision graph of TUC_4C_8 [p, q] nanotube. Then

$$MND(H) = 24p, \quad \text{if } p > 1, q > 1,$$
$$= 20p, \quad \text{if } p > 1, q = 1.$$

Proof: Case 1. Suppose p>1, $q > 1$.
By using equation (4) and Table 3, we obtain

$$MND(H) = \sum_{uv \in E(H)} \left| D_H(u) - D_H(v) \right|$$
$$= |7-7|\, 2p + |7-11|\, 4p + |11-12|\, 8p + |12-12|\, (18pq - 19p)$$
$$= 24p.$$

Case 2. Suppose $p > 1$, $q = 1$.
From equation (4) and by using Table 4, we deduce

$$MND(H) = \sum_{uv \in E(H)} \left| D_H(u) - D_H(v) \right|$$
$$= |7-7|\, 2p + |7-11|\, 4p + |11-11|\, 2p + |11-12|\, 4p + |12-12|\, p$$
$$= 20p.$$

Theorem 13. Let H be the line graph of subdivision of TUC_4C_8 [p, q] nanotube. Then

$$QND(H) = 72p, \quad \text{if } p > 1, q > 1,$$
$$= 68p, \quad \text{if } p > 1, q = 1.$$

Proof: Case 1. Suppose $p > 1$ $q > 1$.
By using equation (5) and Table 3, we obtain

$$QND(H) = \sum_{uv \in E(H)} \left[D_H(u) - D_H(v) \right]^2$$
$$= (7-7)^2\, 2p + (7-11)^2\, 4p + (11-12)^2\, 8p + (12-12)^2\, (18pq - 19p)$$
$$= 72p.$$

Case 2. Suppose $p > 1$, $q = 1$.
From equation (5) and by using Table 4, we have

$$QND(H) = \sum_{uv \in E(H)} \left[D_H(u) - D_H(v) \right]^2$$
$$= (7-7)^2\, 2p + (7-11)^2\, 4p + (11-11)^2\, 2p + (11-12)^2\, 4p + (12-12)^2\, p$$
$$= 68p.$$

Theorem 14. Let H be the line graph of subdivision graph of TUC_4C_8 [p, q] nanotube. Then

$$F_1ND(H,x) = 2px^{98} + 4px^{170} + 8px^{265} + (18pq - 19p)x^{288}, \quad \text{if } p>1, q>1,$$
$$= 2px^{98} + 4px^{170} + 2px^{242} + 4px^{265} + px^{288}, \quad \text{if } p>1, q=1$$

ISSN: 2277-9655
Impact Factor: 5.164
CODEN: IJESS7

Proof: Case 1. Suppose *p* >1, *q* >1.
From equation (6) and by using Table 3, we have

$$F_1ND(H,x)=\sum_{uv\in E(H)} x^{D_H(u)^2+D_H(v)^2}$$

$$=2px^{7^2+7^2}+4px^{7^2+11^2}+8px^{11^2+12^2}+(18pq-19p)x^{12^2+12^2}$$

$$=2px^{98}+4px^{170}+8px^{265}+(18pq-19p)x^{288}$$

Case 2. Suppose *p* > 1, *q* = 1.
By using equation (6) and Table 4, we derive

$$F_1ND(H,x)=\sum_{uv\in E(H)} x^{D_H(u)^2+D_H(v)^2}$$

$$=2px^{7^2+7^2}+4px^{7^2+11^2}+2px^{11^2+11^2}+4px^{11^2+12^2}+px^{12^2+12^2}$$

$$=2px^{98}+4px^{170}+2px^{242}+4px^{265}+px^{288}.$$

Theorem 15. Let *H* be the line graph of subdivision graph of TUC_4C_8 [*p*, *q*] nanotube. Then

$$MND(H,x)=(18pq-17p)x^0+8px^1+4px^4, \quad \text{if } p>1, q>1,$$

$$=5px^0+4px^1+4px^4, \quad \text{if } p>1, q=1.$$

Proof: Case 1. Suppose *p* > 1, *q* > 1.
From equation (7) and by using Table 3, we deduce

$$MND(H,x)=\sum_{uv\in E(H)} x^{|D_H(u)-D_H(v)|}$$

$$=2px^{|7-7|}+4px^{|7-11|}+8px^{|11-12|}+(18pq-19p)x^{|12-12|}$$

$$=(18pq-17p)x^0+8px^1+4px^4.$$

Case 2. Suppose *p* > 1, *q* = 1.
By using equation (7) and Table 4, we derive

$$MND(H,x)=\sum_{uv\in E(H)} x^{|D_H(u)-D_H(v)|}$$

$$=2px^{|7-7|}+4px^{|7-11|}+2px^{|11-11|}+4px^{|11-12|}+p^{|12-12|}$$

$$=5px^0+4px^1+4px^4.$$

Theorem 16. Let *H* be the line graph of subdivision graph of TUC_4C_8 [*p*, *q*] nanotube. Then

$$QND(H,x)=[18pq-17p]x^0+8px^1+4px^{16}, \quad \text{if } p>1, q>1,$$

$$=5px^0+4px^1+4px^{16}, \quad \text{if } p>1, q=1.$$

Proof: Case 1. Suppose *p* > 1, *q* >1.
By using equation (8) and Table 3, we deduce

$$QND(H,x)=\sum_{uv\in E(H)} x^{[D_H(u)-D_H(v)]^2}$$

$$=2px^{(7-7)^2}+4px^{(7-11)^2}+8px^{(11-12)^2}+(18pq-19p)x^{(12-12)^2}$$

ISSN: 2277-9655
Impact Factor: 5.164
CODEN: IJESS7

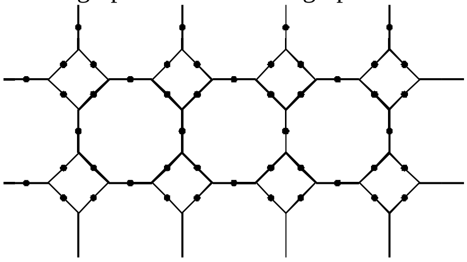
$$=\left(18pq-17p\right)x^{0}+8px^{1}+4px^{16}.$$

Case 2. Suppose $p > 1$, $q = 1$.
From equation (8) and by using Table 4, we derive

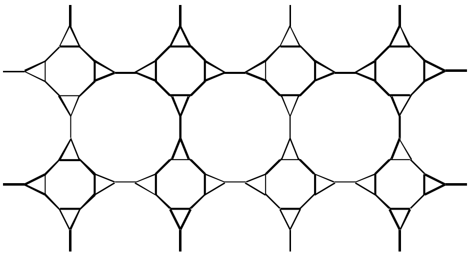
$$QND\left(H,x\right)=\sum_{uv\in E(H)}x^{\left[D_H(u)-D_H(v)\right]^2}$$
$$=2px^{(7-7)^2}+4px^{(7-11)^2}+2px^{(11-11)^2}+4px^{(11-12)^2}+px^{(12-12)^2}$$
$$=5px^{0}+4px^{1}+4px^{16}.$$

4. TUC_4C_8 [p, q] Nanotorus

The line graph of subdivision graph of TUC_4C_8 [4,2] nanotorus is presented in Figure 4(b).



(a)
Subdivision graph of
$TUC_4C_8[p, q]$ nanotorus



(b)
Line graph of subdivision
graph of $TUC_4C_8[p, q]$ nanotorus

Figure 4

The graph of $TUC_4C_8[p, q]$ nanotorus has $4pq$ vertices and $6pq$ edges. By Lemma 1, the subdivision graph of TUC_4C_8 [p, q] nanotorus is a graph with $10pq$ vertices and $12pq$ edges. Thus by Lemma 2, the line graph of subdivision graph of $TUC_4C_8[p, q]$ nanotorus K has $12pq$ vertices and $18pq$ edges. Clearly the degree of each vertex of K is 3. The edge partition based on the degree sum of closed neighborhood vertices of each vertex is as given in Table 5.

Table 5. Edge partition of K

$D_K(u), D_K(v)\backslash uv \in E(K)$	(12, 12)
Number of edges	$18pq$

Theorem 17. Let K the line graph of subdivision graph of TUC_4C_8 [p, q] nanotorus. Then

$$^{m}ND_1(K)=\tfrac{3}{4}pq.$$
$$^{m}ND_2(K)=\tfrac{1}{8}pq.$$
$$F_1ND(K)=5184pq.$$
$$MND(K)=0.$$
$$QND(K)=0.$$
$$F_1ND(K,x)=18pq\,x^{288}.$$
$$MND(K,x)=18pq\,x^{0}.$$
$$QND(K,x)=18pq\,x^{0}.$$

ISSN: 2277-9655
Impact Factor: 5.164
CODEN: IJESS7

Proof: By using definitions and Table 5, we obtain the desired results

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